**XXIV ONPCE: Oficina Nacional de Problemas de Corte e Empacotamento, Planejamento e Programação da Produção e Correlatos**

# Solutions methods for the Hydro Electric Recharge Location Problem

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Alternative fuel vehicles (AFVs) are known to have a more limited range related to standard fuel ones, which increases the necessary amount of recharge stations (RSs) to be installed in any scenario [1], so the Hydro Electric Recharge Location Problem (HERLP) arises from the need to solve and balance the number of RSs necessary to cover the trips between multiple common origin and destination points. An OD pair is considered to be covered if all its candidate paths are also covered. A path is considered to be covered if all its sub-paths are longer than the AFV's range. A candidate path is a path that the AFV is likely to take, i.e., those ones in range from the minimum path until a maximum tolerance.

Figure 1 illustrates a small road network with four origin/destination nodes (A, B, C, and D). Selecting path $DKFIJB$ between D and B and a range of 30, Figure 2 illustrates this path coverage condition, with a length of 70, it requires at least two RSs. However, if both are located between D and K would leave parts of the path with a length greater than 30 without any RS. If we also cover all sub-paths with lengths greater than 30, such as $KFIJ$ requires at least one RS, $IJB$ one, and $KFIJB$ two.

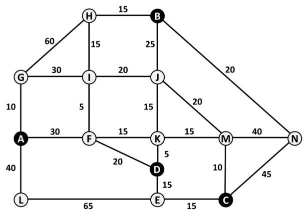

Figure 1: Hypothetical road network.

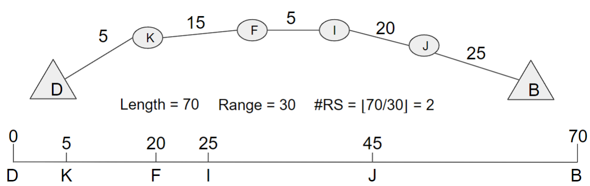


Figure 2: Path coverage example.

We solve this problem with a bi-objective approach since we aim to maximize coverage and minimize the number of RSs installed to any coverage level. The bi-objective problem was solved with heuristic and exact e-constraint algorithms.

In order to cover any OD pair, we need to cover all sub-paths between all probable paths as we need to cover every candidate path. This generates a large number of constraints, so we evaluated static formulations, branch-and-cut (B&C), cut-and-solve (C&S), and a mixed approach in order

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to exactly solve each e-constrained case. For a static formulation, we implemented an algorithm to enumerate all candidate paths, followed by all feasible sub-paths. Besides, for the dynamic algorithms (B&C and C&S), we implemented a separation problem [2] which takes a solution as input and returns a set of violated sub-paths, or validates the input solution as feasible otherwise. Our test instances are based on the publicly available Brazilian transportation road network, where each instance represents a state with the OD pairs indicating cities.

Table 1 shows the results for eight instances, in which we were able to find the optimal Pareto compared against our proposed heuristic in configurations “HXXX”, where the “XXX” is the target number of points. We compare the percentile hyper-volume (HV) and CPU time for each heuristic configuration.

| | H100 | | H050 | | H025 | | H010 | |
|---|---|---|---|---|---|---|---|---|
| Instance | HV (%) | Time (%) | HV (%) | Time (%) | HV (%) | Time (%) | HV (%) | Time (%) |
| AP | 100.00 | 98.84 | 100.00 | 76.92 | 100.00 | 97.45 | 100.00 | 92.24 |
| AM | 100.00 | 59.71 | 100.00 | 68.72 | 88.43 | 22.45 | 81.87 | 12.08 |
| RR | 99.99 | 76.02 | 99.99 | 68.65 | 99.99 | 92.76 | 98.15 | 63.18 |
| AC | 96.03 | 95.00 | 95.51 | 33.79 | 71.91 | 47.44 | 38.39 | 40.40 |
| RO | 99.75 | 75.51 | 99.13 | 36.75 | 97.86 | 16.93 | 93.39 | 5.18 |
| SE | 99.73 | 61.36 | 99.29 | 47.84 | 98.54 | 46.72 | 94.17 | 43.60 |
| AL | 99.76 | 13.46 | 99.39 | 6.89 | 98.24 | 4.84 | 94.02 | 1.18 |
| TO | 99.50 | 2.08 | 98.77 | 1.00 | 97.26 | 0.47 | 92.47 | 0.15 |
| Average | 99.34 | 60.25 | 99.01 | 42.57 | 94.03 | 41.13 | 86.56 | 32.25 |

Table 1: Proportional results obtained with heuristics in relation to the exact method

Table 2 shows the results for 14 instances with fixed minimum coverage and a time limit with multiple solution algorithms. The Full Formulation first calculates all candidate paths, followed by all feasible sub-paths, and solved with our MIP solves which considers it as a static formulation. The B&C starts the MIP solver with no sub-paths, solves a separation problem, and dynamically adds the violated constraints if necessary. The C&S starts a current formulation with no sub-paths, call the MIP solver to solve it with a smaller time limit, applies the separation problem to the best solution found, add the returned sub-paths to the current formulation and restarts the process or stop it if the best solution found is feasible. The C&S+B&C starts with a C&S until a criterion is met, and then, starts a B&C with the final C&S formulation as an initial B&C formulation.

| | C&S+B&C | | | C&S | | | B&C | | | Full Formulation | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Instance | Sol | Gap (%) | Time (s) | Sol | Gap (%) | Time (s) | Sol | Gap (%) | Time (s) | Sol | Gap (%) | Time (s) |
| AP | 14 | 0.00 | 0.25 | 14 | 0.00 | 0.31 | 14 | 0.00 | 5.31 | 14 | 0.00 | 0.69 |
| AM | 90 | 0.00 | 1.69 | 90 | 0.00 | 1.00 | 90 | 0.00 | 5.00 | 90 | 0.00 | 1.75 |
| RR | 13 | 0.00 | 0.63 | 13 | 0.00 | 1.22 | 13 | 0.00 | 6.92 | 13 | 0.00 | 1.82 |
| AC | 15 | 0.00 | 0.89 | 15 | 0.00 | 0.38 | 15 | 0.00 | 2.18 | 15 | 0.00 | 1.26 |
| RO | 16 | 0.00 | 19.96 | 16 | 0.00 | 52.26 | 16 | 0.00 | 250.22 | 16 | 0.00 | 58.44 |
| SE | 9 | 0.00 | 31.39 | 9 | 0.00 | 37.96 | 9 | 0.00 | 637.98 | 9 | 0.00 | 71.34 |
| AL | 17 | 0.00 | 299.07 | 17 | 0.00 | 4236.92 | 17 | 0.00 | 9354.92 | 17 | 0.00 | 3743.56 |
| TO | 72 | 0.00 | 115.03 | 72 | 0.00 | 624.35 | 72 | 0.00 | 2755.13 | 72 | 0.00 | 1536.24 |
| PA | 103 | 18.39 | 14400.00 | 108 | 21.35 | 14400.00 | 110 | 29.41 | 14400.00 | - | - | 14400.00 |
| MA | 58 | 3.57 | 14400.00 | 61 | 8.93 | 14400.00 | 64 | 16.36 | 14400.00 | 68 | 23.64 | 14400.00 |
| MS | 53 | 0.00 | 5877.77 | 53 | 0.00 | 14385.09 | 56 | 9.80 | 14400.00 | 61 | 27.08 | 14400.00 |
| PB | 38 | 0.00 | 12774.96 | 38 | 5.56 | 14400.00 | 38 | 8.57 | 14400.00 | 91 | 167.65 | 14400.00 |
| RN | 32 | 0.00 | 5952.70 | 33 | 6.45 | 14400.00 | 35 | 12.90 | 14400.00 | 41 | 41.38 | 14400.00 |
| ES | 29 | 0.00 | 3728.07 | 30 | 7.14 | 14400.00 | 32 | 18.52 | 14400.00 | - | - | 14400.00 |

Table 2: Results for a coverage level of 90% and a time limit of 14400 seconds.

# References


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